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# The Role of Coherence in Time Delay Estimation.

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(10) G. Clifford Carter
Special Projects Department

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This paper investigated methods for passive estimation of the bearing to a slowly moving acoustically radiating source. The mathematics for the solution to such a problem is analogous to estimating the time delay (or group delay) between two time series. Since the estimation of time delay is intimately related to the coherence between two time series, a summary of the properties of coherence is presented.



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# The Role of Coherence in Time Delay Estimation

$$C_{ab}(f) = \frac{\left|G_{ab}(f)\right|^2}{G_{aa}(f)\,G_{bb}(f)}$$

$$0 \leq C_{ab}(f) \leq 1, \forall f$$

SLIDE 1

(SLIDE 1)

THE TERM COHERENCE HAS SEVERAL DIFFERENT MEANINGS AND INDEED DEFINITIONS. THE ONE WE USE HERE IS THE MAGNITUDE SQUARED OF THE COEFFICIENT OF COHERENCY DEFINED BY WEINER IN 1930. IN PARTICULAR, FOR OUR PURPOSES HERE, WE DEFINE THE COHERENCE BETWEEN TWO STATIONARY RANDOM PROCESSES A AND B AS THE MAGNITUDE SQUARED OF THE CROSS POWER SPECTRUM DIVIDED BY THE PRODUCT OF THE TWO AUTO POWER SPECTRA. THE COHERENCE IS A FUNCTION OF FREQUENCY AND HAS THE USEFUL PROPERTY THAT IT LIES BETWEEN ZERO AND UNITY. IT IS, IN EFFECT, A NORMALIZED CROSS SPECTRAL DENSITY THAT, IN SOME SENSE, MEASURES THE EXTENT TO WHICH TWO RANDOM PROCESSES ARE SIMILAR. FOR EXAMPLE, TWO UNCORRELATED RANDOM PROCESSES ARE ALSO INCOHERENT; THAT IS, THE COHERENCE IS ZERO BETWEEN UNCORRELATED PROCESSES. FURTHER, THE COHERENCE BETWEEN TWO LINEARLY RELATED PROCESSES IS UNITY.

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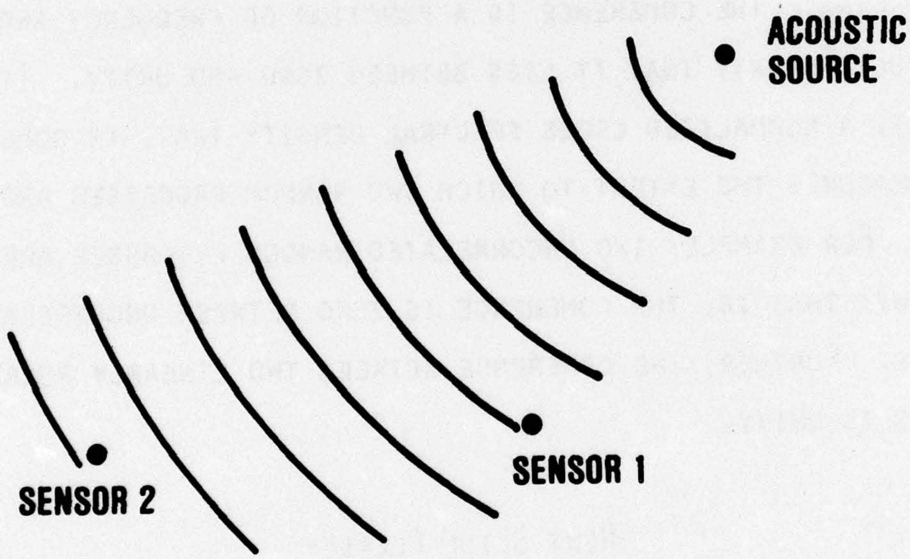


SLIDE 2

(SLIDE 2)

THE PHYSICAL PROBLEM THAT MOTIVATES THIS RESEARCH IS A DESIRE TO PASSIVELY ESTIMATE GEOGRAPHICAL INFORMATION ABOUT THE STATE OF AN ACOUSTIC SOURCE. IN THE DEVELOPMENT HERE AN ACOUSTIC POINT SOURCE RADIATES SPHERICAL WAVES, RECEIVED, FIRST, AT ONE SENSOR AND SOME DELAYED TIME LATER, AT A SECOND SENSOR. THE SOURCE IS ASSUMED STATIONARY FOR THE OBSERVATION PERIOD AND THE SENSOR SEPARATION IS ASSUMED KNOWN. EACH RECEIVED WAVEFORM IS OBSERVED IN THE PRESENCE OF UNCORRELATED NOISE. THE PROBLEM WE ADDRESS IS HOW TO ESTIMATE THE TRAVEL TIME OF THE WAVEFRONT OR TIME DELAY FROM ONE SENSOR TO THE NEXT.

THE IMPORTANCE OF OBTAINING A GOOD TIME DELAY ESTIMATE IS THAT IT CAN BE USED TO FIX THE SOURCE LOCATION ON A HYPERBOLIC LOCUS OF POINTS. OF COURSE, IF WE HAVE THREE SENSORS WE CAN ESTIMATE TWO TIME DELAYS AND THE INTERSECTING HYPERBOLIC CURVES CAN BE USED TO ESTIMATE SOURCE POSITION.

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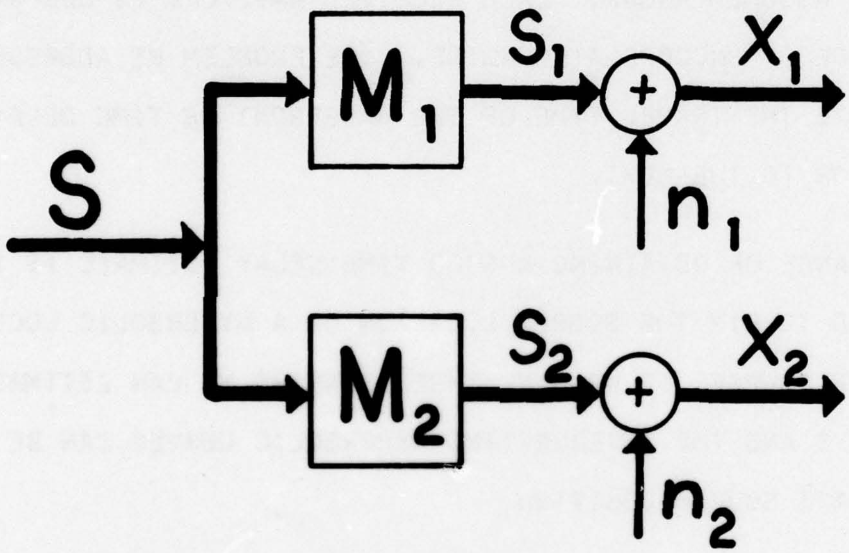
# GENERAL CASE




# SPECIFIC CASE

$$X_1(t) = S(t) + n_1(t)$$

$$X_2(t) = \alpha S(t+D) + n_2(t)$$

SLIDE 3

(SLIDE 3)

IN THE GENERAL CASE WE CAN MODEL THE ACOUSTIC SOURCE PROPAGATION AND NOISE CORRUPTED RECEPTION AS SHOWN HERE. IN PARTICULAR, WE TREAT THE PATH FROM THE SOURCE TO EACH RECEIVER AS A LINEAR TIME INVARIANT FILTER. THE RECEIVED SIGNALS X ONE AND X TWO CONSIST OF THE FILTER OUTPUTS PLUS NOISE.

A SPECIAL CASE OF THIS MODEL IS SHOWN ON THE BOTTOM OF THE SLIDE. THE FIRST RECEIVED WAVEFORM CONSISTS OF SIGNAL PLUS NOISE. THE SECOND RECEIVED WAVEFORM CONSISTS OF AN ATTENUATED AND DELAYED SIGNAL IN THE PRESENCE OF NOISE. THE MATHEMATICAL PROBLEM WE ADDRESS IS: HOW TO BEST ESTIMATE THE TIME DELAY OR EQUIVALENTLY SOURCE BEARING. FURTHER WE ARE CONCERNED WITH THE ROLE OF COHERENCE IN THIS PROCESS.

FOR ANALYTIC PURPOSES WE TREAT THE NOISE AS STATIONARY AND UNCORRELATED. LATER WE MAKE AN IMPLICIT ASSUMPTION THAT THE NOISE IS NORMAL (GAUSSIAN).

-NEXT SLIDE PLEASE-

$$\frac{G_{s_i s_i}(f)}{G_{n_i n_i}(f)} = \frac{C_{sx_i}(f)}{1 - C_{sx_i}(f)}, \quad i = 1, 2;$$

$$C_{12}(f) \equiv C_{x_1 x_2}(f) = C_{sx_1}(f)\, C_{sx_2}(f)$$

SLIDE 4

(SLIDE 4)

FOR THE GENERAL MODEL WE CAN SHOW THAT THE RECEIVED SIGNAL-TO-NOISE RATIO IS A FUNCTION OF ONLY THE COHERENCE BETWEEN THE SOURCE AND THE RECEIVER. INDEED, RELATIVE TO THE SENSOR NOISE POWER, THE AMOUNT OF POWER RECEIVED FROM THE SOURCE AFTER IT HAS BEEN ATTENUATED BY ACOUSTIC TRANSMISSION THROUGH THE OCEAN MEDIUM IS DESCRIBED BY THE SOURCE-TO-SENSOR COHERENCE DIVIDED BY ONE MINUS SOURCE-TO-SENSOR COHERENCE. MOREOVER, THE COHERENCE BETWEEN THE TWO RECEIVED WAVEFORMS CAN<u>NOT</u> EXCEED THE COHERENCE BETWEEN THE SOURCE AND ANY SENSOR; THIS IS TRUE WHEN THE OCEAN MEDIUM IS MODELED AS A LINEAR TIME INVARIANT FILTER.

MORE SPECIFICALLY, THE COHERENCE BETWEEN THE TWO RECEIVED WAVEFORMS IS EQUAL TO THE PRODUCT OF THE COHERENCES BETWEEN THE SOURCE AND EACH OF THE RECEIVED WAVEFORMS.

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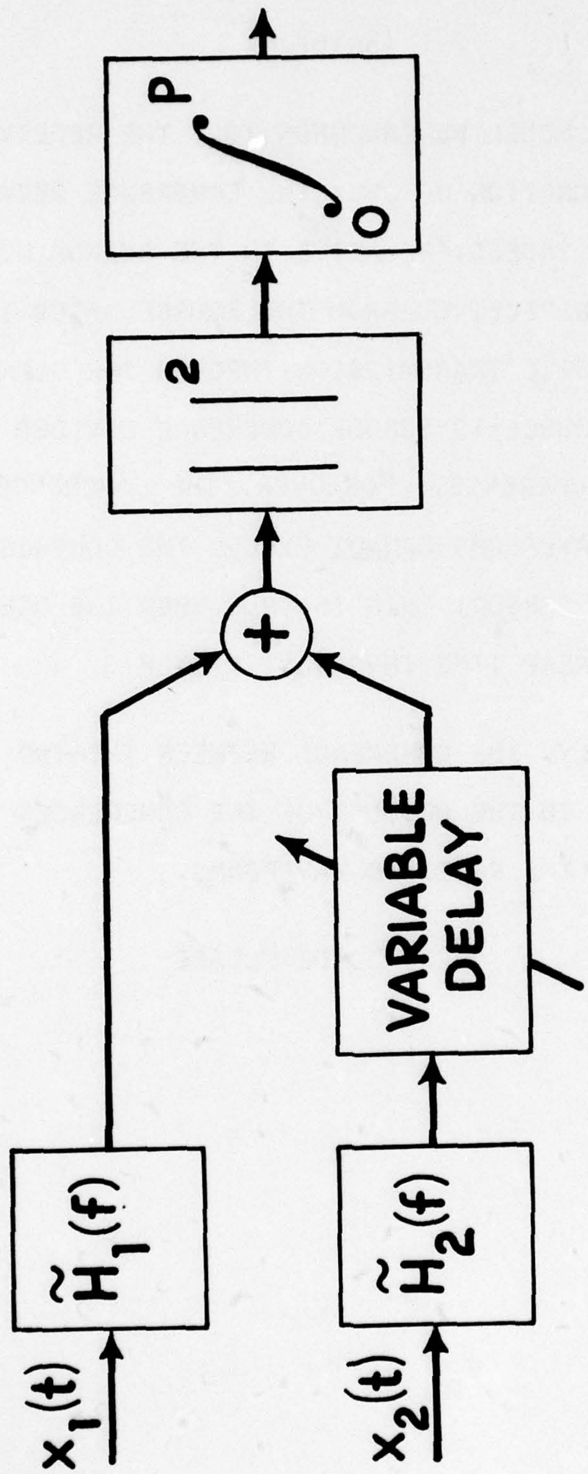


SLIDE 5

(SLIDE 5)

UNDER STANDARD ASSUMPTIONS, NAMELY, THAT THE TWO RECEIVED WAVEFORMS, X ONE AND X TWO, ARE JOINTLY STATIONARY, NORMAL (GAUSSIAN) RANDOM PROCESSES AND THAT THE OBSERVATION TIME P IS LARGE, THE MAXIMUM LIKELIHOOD ESTIMATE OF TIME DELAY CAN BE DERIVED. THE MAXIMUM LIKELIHOOD,OR ML,ESTIMATE OF TIME DELAY CAN BE INSTRUMENTED INONE OF TWO WAYS. SHOWN HERE IS ONE REALIZATION. THE FIRST RECEIVED WAVEFORM IS FILTERED BY H ONE TILDE, AND THE SECOND RECEIVED WAVEFORM IS FILTERED BY H TWO TILDE AND DELAYED. THE FILTERS MUST HAVE IDENTICAL PHASE RESPONSES. THE FILTER OUTPUTS ARE SUMMED, SQUARED, AND INTEGRATED AS SHOWN. THE HYPOTHESIZED VARIABLE DELAY THAT MAXIMIZES THIS SYSTEM OUTPUT IS THE MAXIMUM LIKELIHOOD ESTIMATE OF TIME DELAY. THE SPECIFIC FILTER CHARACTERISTICS DEPEND ON THE SIGNAL AND NOISE SPECTRA, WHICH MUST BE KNOWN OR ESTIMATED.

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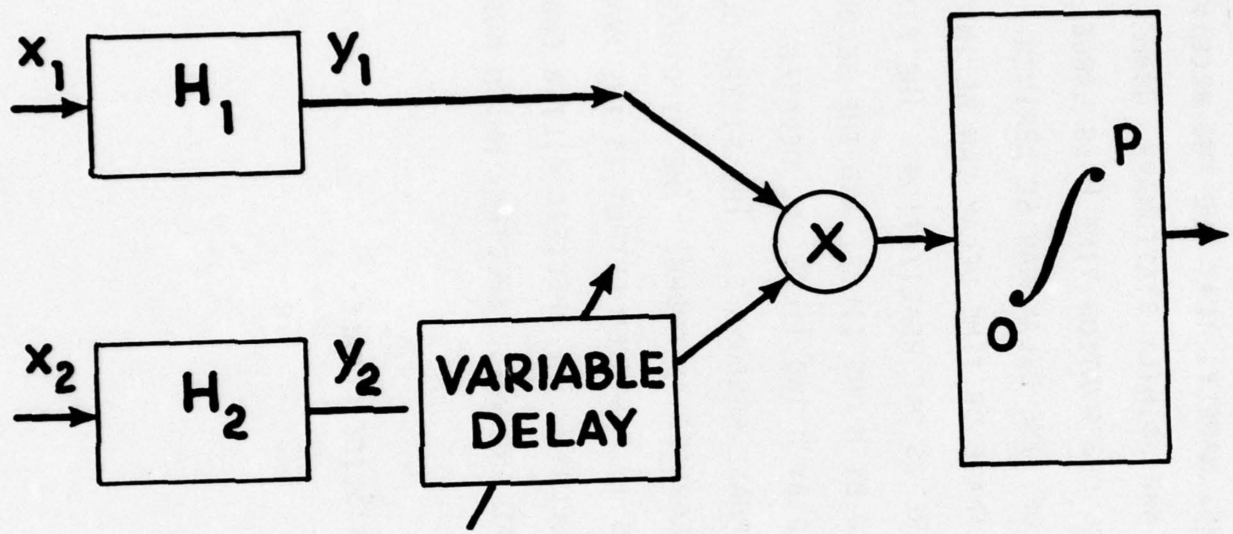


$$W_g(f) \triangleq H_1(f)\,H_2^*(f)$$

$$R^{g}_{x_1 x_2}(\tau) = \int_{-\infty}^{\infty} W_g(f)\, G_{x_1 x_2}(f)\, e^{j2\pi f \tau}\, df$$

SLIDE 6

(SLIDE 6)

ANOTHER REALIZATION FOR THE MAXIMUM LIKELIHOOD ESTIMATE OF TIME DELAY IS A SPECIAL CASE OF THE GENERALIZED CROSSCORRELATION PROCESSOR. IN THIS PROCESSOR THE FIRST RECEIVED WAVEFORM IS FILTERED BY H ONE AND THE SECOND RECEIVED WAVEFORM IS FILTERED BY H TWO, DELAYED, MULTIPLIED, AND INTEGRATED AS SHOWN ON THE DIAGRAM AT THE TOP OF THE SLIDE. BY PROPER CHOICE OF THE GENERAL WEIGHTING FUNCTION, W, WHICH IS THE PRODUCT OF H ONE AND H TWO CONJUGATE, WE CAN ACHIEVE THE MAXIMUM LIKELIHOOD ESTIMATE FOR TIME DELAY. HOWEVER, WE CAN ALSO ACHIEVE A GENERALIZED CROSSCORRELATION FUNCTION, R OF TAU, BY MULTIPLYING THE CROSS SPECTRUM, G, BETWEEN THE TWO RECEIVED WAVEFORMS, BY THE GENERAL WEIGHTING, W, AND COMPUTING THE FOURIER TRANSFORM AS INDICATED ON THE BOTTOM OF THE SLIDE.

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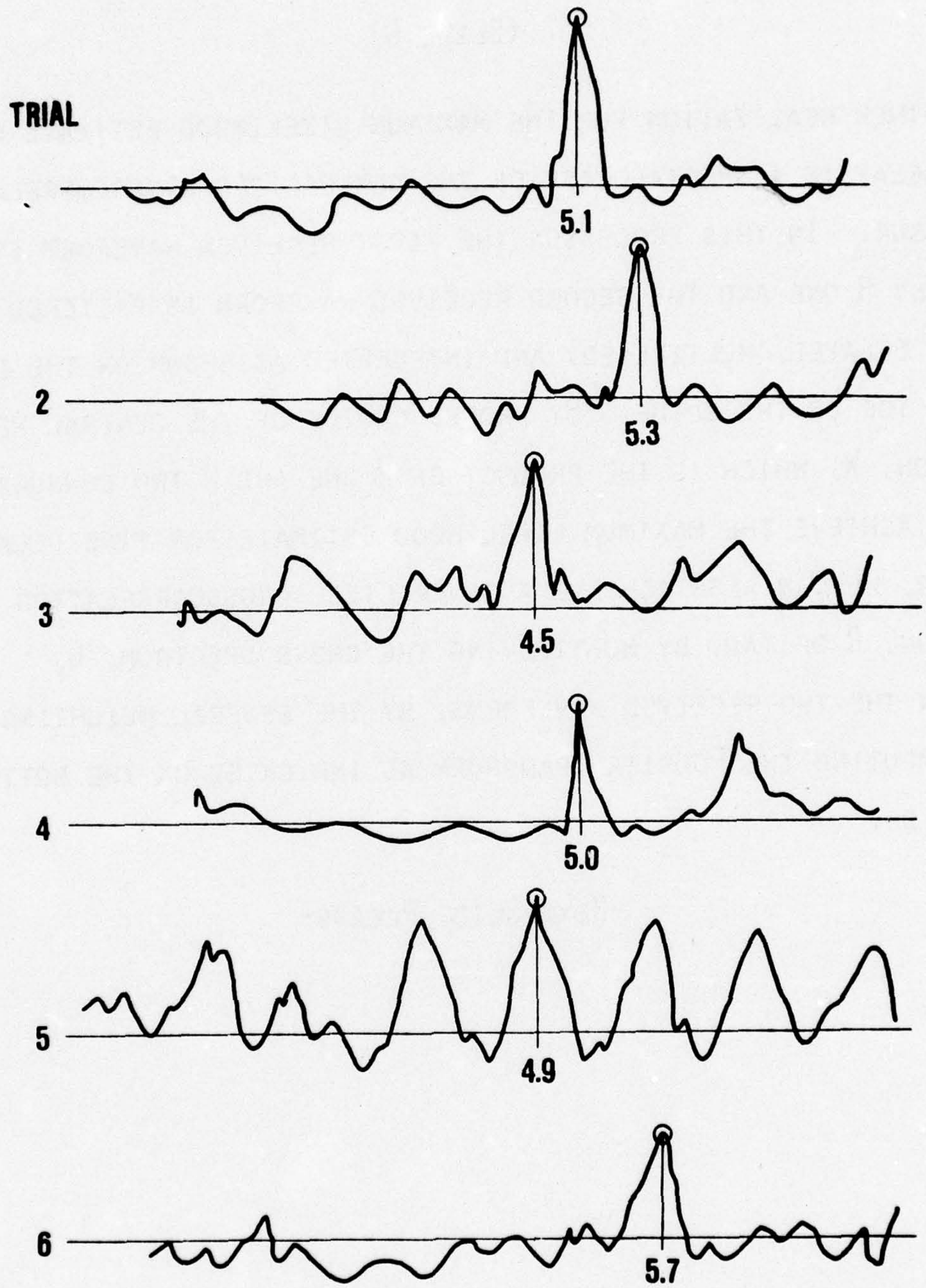


SLIDE 7

(SLIDE 7)

IF WE ESTIMATED THE GENERALIZED CROSSCORRELATION FUNCTION FOR SIX DIFFERENT TRIALS, THE PEAK OF THE FUNCTION MIGHT VARY AS A FUNCTION OF TRIAL. WE HAVE ACTUALLY IMPLEMENTED THE TECHNIQUE FOR SEVERAL EXAMPLE CASES ON THE UNIVAC 1108. BASED ON OUR EXPERIMENTAL RESULTS, WE SPECULATE THAT A TYPICAL GENERALIZED CROSSCORRELATION FUNCTION MIGHT PEAK, AS INDICATED IN THE HYPOTHETICAL TRIALS SKETCHED HERE. IN PARTICULAR, THE ABSCISSA VALUE OF THE PEAK LOCATION, THAT IS, THE ESTIMATE OF TIME DELAY, HAS A CERTAIN AMOUNT OF VARIATION. NOTICE ALSO IN TRIAL NUMBER 5, NEXT TO THE BOTTOM PLOT, THAT A NUMBER OF AMBIGUOUS PEAKS CAN ARISE IN ADDITION TO THE LOCAL VARIATION OF THE TIME DELAY ESTIMATE. THE AMBIGUITY PROBLEM IS NOT TREATED IN THIS WORK. THE PROBLEM OF COMPUTING THE VARIANCE OF THE TIME DELAY ESTIMATE IS A DIFFICULT ONE IN WHICH ONE IS PUZZLED HOW TO PROCEED. HOWEVER, IF WE COULD COUNT THE NUMBER OF PEAKS THAT OCCURRED AT EACH OF SEVERAL ABSCISSA VALUES, THEN WE COULD PLOT A FREQUENCY DISTRIBUTION OF THE PEAK LOCATION. FROM THIS DISTRIBUTION WE CAN OBTAIN THE VARIANCE OF THE TIME DELAY ESTIMATE.

-NEXT SLIDE PLEASE-

# VARIANCE OF DELAY ESTIMATE

$$\frac{\int_{-\infty}^{\infty} df \left|W_g(f)\right|^2 (2\pi f)^2 G_{11}(f) G_{22}(f) \left[1 - C_{12}(f)\right]}{P \left[\int_{-\infty}^{\infty} df (2\pi f)^2 \left|G_{12}(f)\right| \cdot \left|W_g(f)\right|\right]^2}$$

SLIDE 8

(SLIDE 8)

THE VARIANCE OF THE TIME DELAY ESTIMATE IS A COMPLICATED FUNCTION OF SEVERAL PARAMETERS. IT DEPENDS ON THE LENGTH OF THE OBSERVATION TIME, P, THE GENERAL WEIGHTING FUNCTION, W, THE AUTO-SPECTRAL DENSITIES OF THE TWO RECEIVED WAVEFORMS, AND THE MAGNITUDE CROSS SPECTRUM BETWEEN THE TWO RECEIVED WAVEFORMS. IT ALSO DEPENDS ON THE COHERENCE, C, DEFINED EARLIER AS THE MAGNITUDE SQUARED CROSS SPECTRUM DIVIDED BY THE PRODUCT OF THE TWO AUTO-SPECTRAL DENSITIES. RECALL THE COHERENCE IS GREATER THAN OR EQUAL TO ZERO AND IS LESS THAN OR EQUAL TO UNITY. WHEN THE OBSERVATION TIME IS LARGE OR THE COHERENCE IS NEAR UNITY, THE VARIANCE IS GENERALLY QUITE LOW AND YOU CAN DO WELL IN SPITE OF THE WEIGHTING SELECTED. OF COURSE, AN IMPORTANT ROLE TO BE PLAYED BY THE EXPRESSION HERE IS TO EVALUATE HOW DIFFERENT PROCESSORS COMPARE WITH ONE ANOTHER. ANOTHER IMPORTANT USE OF THIS EXPRESSION IS IF ONE KNOWS THEORETICALLY THE BEST WEIGHTING FUNCTION TO APPLY, BUT APPLIES AN INCORRECT OR SUBOPTIMUM WEIGHTING, THEN THE VARIANCE OF THE SUBOPTIMUM DELAY ESTIMATOR CAN BE EVALUATED.

-NEXT SLIDE PLEASE-

# MINIMUM VARIANCE

$$\left[ 2P \int_0^{\infty} df \, (2\pi f)^2 \frac{C_{12}(f)}{1 - C_{12}(f)} \right]^{-1}$$

FOR

$$W_{ML}(f) = \frac{C_{12}(f)}{\left| G_{12}(f) \right| \left[ 1 - C_{12}(f) \right]}$$

SLIDE 9

(SLIDE 9)

THE MINIMUM VARIANCE FOR ANY TIME DELAY ESTIMATION SCHEME CAN BE OBTAINED FROM THE CRAMÉR RAO LOWER BOUND. AS SHOWN HERE, IT IS A FUNCTION OF ONLY TWO PARAMETERS: THE OBSERVATION TIME P AND THE COHERENCE BETWEEN THE TWO RECEIVED WAVEFORMS. AS P IS INCREASED THE VARIANCE DROPS; FURTHER, AS THE COHERENCE C TENDS TOWARD UNITY THE TERM C OVER ONE MINUS C SQUARED TENDS TOWARDS INFINITY. THUS, AS THE COHERENCE OR C TENDS TOWARDS UNITY, THE VARIANCE TENDS TOWARDS ZERO. HOWEVER, THE COHERENCE IS NOT UNDER OUR CONTROL. THE FACTORS WHICH WE CAN CONTROL ARE THE OBSERVATION TIME P AND THE WEIGHTING. THE MINIMUM VARIANCE IS ACHIEVED FOR THE MAXIMUM LIKELIHOOD WEIGHTING FUNCTION GIVEN BY C OVER ONE MINUS C TIMES THE MAGNITUDE CROSS SPECTRUM.

-NEXT SLIDE PLEASE-

$$\frac{\hat{G}_{12}(f)}{\left|\hat{G}_{12}(f)\right|} \cdot \frac{\hat{C}_{12}(f)}{\left[1-\hat{C}_{12}(f)\right]}$$

$$e^{j\hat{\theta}(f)} \cdot \frac{\hat{C}_{12}(f)}{\left[1-\hat{C}_{12}(f)\right]}$$

SLIDE 10

(SLIDE 10)

THE MAXIMUM LIKELIHOOD WEIGHTING MULTIPLIES THE ESTIMATED CROSS SPECTRUM TO YIELD A SINGLE FUNCTION TO BE FOURIER TRANSFORMED. IN GENERAL, WHEN THE TRUE VALUES OF COHERENCE AND MAGNITUDE CROSS SPECTRUM ARE UNKNOWN, THEY MUST BE ESTIMATED. ESTIMATES ARE INDICATED BY HATS. WHEN SPECTRAL ANALYSIS IS USED TO YIELD ESTIMATES IN PLACE OF THE TRUE QUANTITIES, THE FUNCTION TO BE FOURIER TRANSFORMED IS INDICATED ON THIS SLIDE. THE CROSS SPECTRUM OVER THE MAGNITUDE CROSS SPECTRUM CAN BE THOUGHT OF AS E TO THE MINUS J PHASE. IN PARTICULAR, NOTE THAT THE WEIGHTING EMPHASIZES THE PHASE OF THE ESTIMATED CROSS SPECTRUM IN THOSE FREQUENCY BANDS WHERE THE COHERENCE IS HIGH. ONE WOULD EXPECT THE ESTIMATED PHASE OF THE CROSS SPECTRUM TO PLAY AN IMPORTANT ROLE IN TIME DELAY ESTIMATION, SINCE THE SLOPE OF THE PHASE IS A MEASURE OF THE TIME DELAY. WE CAN SEE THIS BY NOTING THAT THE PHASE SLOPE IS MEASURED IN RADIANS, DIVIDED BY RADIANS PER SECOND, OR SECONDS. OF COURSE, THE PHASE ESTIMATES WILL BE NOISY IN THOSE FREQUENCY BANDS WHERE THE COHERENCE IS LOW SO WE WILL EMPHASIZE THE PHASE IN THOSE BANDS WHERE THE COHERENCE IS HIGH.

-NEXT SLIDE PLEASE-

# SUMMARY

- ACOUSTIC SOURCE
- TIME DELAY MODEL
- DERIVED ML TIME DELAY ESTIMATE
- DERIVED CRAMÉR RAO LOWER ROUND
- DERIVED THE VARIANCE FOR ANY GCC
- SHOWN ML ESTIMATE IS MINIMUM VAR
- IMPLEMENTED RESULTS
- APPLICATIONS TO ESTIMATING SOURCE POSITION

SLIDE 11

(SLIDE 11)

IN SUMMARY, THE PHYSICAL PROBLEM MOTIVATING THIS RESEARCH IS A DESIRE TO ESTIMATE POSITIONAL INFORMATION ABOUT AN ACOUSTIC SOURCE. WE HAVE PROPOSED A TIME DELAY MODEL AND DERIVED THE MAXIMUM LIKELIHOOD ESTIMATE FOR TIME DELAY. ADDITIONALLY WE HAVE DERIVED THE CRAMÉR RAO LOWER BOUND ON THE VARIANCE OF THE TIME DELAY ESTIMATE. SUBSEQUENTLY WE HAVE DERIVED AN EXPRESSION FOR THE VARIANCE OF THE TIME DELAY ESTIMATE FOR ANY GENERALIZED CROSS-CORRELATION PROCESSOR. WE HAVE SHOWN THAT THE MAXIMUM LIKELIHOOD ESTIMATE OF TIME DELAY ACHIEVES THE CRAMÉR RAO LOWER BOUND AND IS THEREFORE MINIMUM VARIANCE; AS SUCH THE PROPOSED TECHNIQUE IS THE BEST PROCESSING THAT CAN BE DONE TO ESTIMATE TIME DELAY OR, EQUIVALENTLY, TO ESTIMATE THE HYPERBOLIC LOCUS OF POINTS ON WHICH THE ACOUSTIC SOURCE IS LOCATED. THERE IS NO BETTER TECHNIQUE. WE HAVE IMPLEMENTED THE RESULTS IN AN APPROXIMATE METHOD BY SUBSTITUTING ESTIMATED MAXIMUM LIKELIHOOD WEIGHTING IN PLACE OF TRUE WEIGHTING AND FOUND THAT THE TECHNIQUE WORKS ON A LARGE SCALE DIGITAL COMPUTER. OF COURSE, THE ABILITY TO LOCATE A SOURCE ON A HYPERBOLIC LOCUS OF POINTS SUGGESTS THAT, WITH THREE SENSORS, INTERSECTING HYPERBOLIC CURVES CAN BE USED TO ESTIMATE SOURCE POSITION.

Additional references not given in the conference proceedings include my recently competed Ph.D. thesis and an article on generalized correlation processing that has just appeared in the August IEEE Transactions on Acoustics Speech and Signal Processing.

Proceedings Reprint

# THE ROLE OF COHERENCE IN TIME DELAY ESTIMATION

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ABSTRACT. This paper investigates methods for passive estimation of the bearing to a slowly moving acoustically radiating source. The mathematics for the solution to such a problem is analogous to estimating the time delay (or group delay) between two time series. Since the estimation of time delay is intimately related to the coherence between two time series, a summary of the properties of coherence is presented.

The maximum likelihood (ML) estimate of time delay (under jointly stationary Gaussian assumptions) is presented. The explicit dependence of time delay estimates on coherence is evident in the estimator realization in which the two time series are prefiltered (to accentuate frequency bands according to the strength of the coherence) and subsequently crosscorrelated. The hypothesized delay at which the generalized crosscorrelation (GCC) function peaks is the time delay estimate. The variance of the time delay estimate is presented and discussed.

INTRODUCTION. An acoustic source whose signal, $s(t)$, is transmitted through the ocean medium and received in the presence of additive noise can be characterized by

$$x_i(t) = s_i(t) + n_i(t) \quad , \; i = 1,2 \tag{1}$$

For the main purposes of this paper $s_1(t)=s(t)$, $s_2(t)=\alpha s(t+D)$ and we desire to present an ML estimator for the time delay D. The delay parameter can be used, in a nondispersive medium with known speed of transmission, to estimate the bearing to an acoustic source (relative to the sensor baseline) or, more generally, to

estimate a hyperbolic "line" of position. Since the final result depends heavily on the coherence between $x_1$ and $x_2$, we precede the development with a concise review of the properties of the coherence function and of results that bear directly on the estimation of time delay.

THEORY OF COHERENCE. For any two jointly stationary random processes $x_1$ and $x_2$, the coefficient of coherency or the complex coherence has been defined by Weiner (1930) as the ratio

$$\frac{G_{x_1x_2}(f)}{\sqrt{G_{x_1x_1}(f)\, G_{x_2x_2}(f)}}$$

where $G_{x_1x_2}(f)$ is the cross power spectral density function between $x_1$ and $x_2$, and $G_{x_ix_i}(f)$, i=1,2 are the auto power spectral density functions at frequency, f.

The magnitude-squared coherence (MSC) or simply the coherence is defined by (see, for example, Carter, Knapp and Nuttall (1973))

$$C_{x_1x_2}(f) = \frac{\left|G_{x_1x_2}(f)\right|^2}{G_{x_1x_1}(f)\, G_{x_2x_2}(f)} \tag{2}$$

A useful property of the MSC is

$$0 \le C_{x_1x_2}(f) \le 1$$

provided the autospectra are positive (in particular non zero).

In order to attach some physical significance to what the coherence measures, consider that the ocean medium operators $M_1$ and $M_2$ are linear time-invariant filters. Thus $s_1(t)$ and $s_2(t)$ in equation (1) are the respective outputs of filters $M_1(f)$ and $M_2(f)$ when excited by source s(t). When the noise, $n_i(t)$, is uncorrelated with the signal, s(t), at the i-th sensor, the ratio of the received signal power at the output of the ocean channel to the corruptive noise power depends on the coherence between the source and the sensor. Specifically, from Carter, Knapp, and Nuttall (1973)

$$\frac{G_{s_i s_i}(f)}{G_{n_i n_i}(f)} = \frac{C_{sx_i}(f)}{1-C_{sx_i}(f)}, \quad i = 1,2 \tag{3}$$

That is, the received signal-to-noise ratio (SNR) at the i-th sensor depends on the coherence between the source and the received waveform. This result has been expressed by Carter and Knapp (1976) more compactly as

$$C_{x_1 x_2}(f) = C_{sx_1}(f)\, C_{sx_2}(f) \tag{4}$$

These results apply only to the case where the medium can be accurately modeled by linear time-invariant filters corrupted by uncorrelated additive noise.

RESULTS. For the purpose of obtaining an ML estimate of delay, certain assumptions are required. In particular, for a signal emanating from a nearfield source and monitored in the presence of noise at two spatially separated sensors we require in equation (1) that $s_1(t) = s(t)$ and $s_2(t) = \alpha s(t+D)$. Further, we require that $\alpha$ is real and $s(t)$, $n_1(t)$, and $n_2(t)$ are real, jointly stationary, Gaussian random processes. Source $s(t)$ and noises, $n_1(t)$ and $n_2(t)$ are assumed to be mutually uncorrelated.

An estimated value of D is the hypothesized value $\tau$ that maximizes the generalized crosscorrelation (GCC) function defined by

$$\hat{R}(\tau) = \int_{-\infty}^{\infty} \hat{G}_{x_1 x_2}(f) W(f) e^{j2\pi f\tau} df. \tag{5}$$

For $x_1(t)$ and $x_2(t)$ real, the ML estimator requires a particular weighting,

$$W(f) = H_1(f) H_2^*(f) = \frac{C_{x_1 x_2}(f)}{\left|G_{x_1 x_2}(f)\right| \left[1 - C_{x_1 x_2}(f)\right]} \tag{6}$$

A complete derivation is given by Carter (1976).

Note from equation (6) that for the ML estimate of delay that W(f) is real. The ML estimator is virtually equivalent to one proposed by Hannan and Thomson (1973). The ML estimator can be achieved by shaping $x_1(t)$ with filter $H_1(f)$ and $x_2(t)$ with filter $H_2(f)$ crosscorrelating the filter outputs, and observing what hypo-

thesized value of delay achieves a maximum.

The estimator can also be achieved by other methods. For example, Hahn (1975), Carter and Knapp (1976) and Carter (1976) present a method of filtering and summing the outputs, squaring and averaging in order to estimate the delay D. The processor could also be realized as a number of "best" estimates of D for a variety of frequencies. The ML estimate is then achieved by performing a weighted average across frequency. For example, Clay, Hinich and Shaman (1973) develop ML estimates of bearing (analogous to delay) for each of a number of different frequencies. To obtain a single estimate of source bearing, these individual estimates should then be combined with weighting dependent upon the particular underlying signal and noise characteristics.

The role of coherence in the weighting used for ML estimation of D is specified in equation (6). Note that those values of coherence near unity are most important; conversely, in those frequency bands where there is no source signal power (hence, where the received waveforms are incoherent), the delay estimate, as would be expected, receives no weight. The ML estimator is actually a function of more fundamental spectral measurements than those specified in equation (6). However, expressing the processor in more fundamental but unnormalized quantities can make interpretation more difficult, though equally correct.

The ML weighting agrees with MacDonald and Schultheiss (1969), and Hahn (1975) under specific conditions (including when there are two sensors and no attenuation).

VARIANCE OF GENERAL TIME DELAY ESTIMATORS. The variance of the time delay estimate in the neighborhood of the true delay for general weighting function W(f) is given by

$$\mathrm{Var}\left[\hat{D}-D\right]=\frac{\int_{-\infty}^{\infty}\left|W(f)\right|^{2}(2\pi f)^{2}G_{x_1x_1}(f)G_{x_2x_2}(f)\left[1-C_{12}(f)\right]df}{P\left[\int_{-\infty}^{\infty}(2\pi f)^{2}\left|G_{x_1x_2}(f)\right|W(f)df\right]^{2}} \tag{7}$$

where P is the observation period (in seconds). From equations (6) and (7), the variance of the ML processor is

$$\mathrm{Var}^{ML}\left[\hat{D}-D\right]=\left[2P\int_{0}^{\infty}\frac{(2\pi f)^{2}\,C_{12}(f)}{1-C_{12}(f)}\,df\right]^{-1} \tag{8}$$

The ML processor achieves the Cramér-Rao lower bound (see Carter (1976)). Therefore, the ML processor achieves a variance less than or equal to that provided by other correlation processors.

These results for variance can be related to MacDonald and Schultheiss (1969) as follows. Define the bearing to an acoustic source, as in Nuttall, Carter and Montavon (1974)

$$\phi = \arc\cos \frac{\xi D}{d} \tag{9}$$

where $\xi$ is the speed of sound in the nondispersive medium and d is the sensor separation. Consider the case where the estimated D equals the true delay plus a perturbation. By a Taylor series expansion,it follows for the bearing error defined by the difference between the true bearing and the estimated bearing that the standard deviation of the bearing error is given by (Carter (1976)):

$$\left[\mathrm{Var}\,(\hat{\phi}-\phi)\right]^{1/2} = \frac{\xi}{d \sin \phi}\left[\mathrm{Var}\,(\hat{D}-D)\right]^{1/2} \tag{10}$$

The term d sin $\phi$ can be viewed as the effective array length (sensor separation) physically steered at the source.
The combining of equations (8) and (10) suggests that, in order to reduce the variance of the bearing estimate, the observation period and the sensor separation should be made as large as possible. This agrees with one's intuition and the results of MacDonald and Schultheiss (1969). Further, the fact that equation (10) depends on the effective array length physically steered toward the source suggests the desirability of sensor mobility to maximize sin $\phi$ when d is limited.

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